

Vectors

1. $\mathbf{a} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -4 \\ 0 \\ 2 \end{pmatrix}$.

Calculate (a) $|\mathbf{b}|$ (b) $|2\mathbf{a} - \mathbf{b}|$ (c) $|3(\mathbf{a} + \mathbf{b})|$

2. (a) Find the magnitude of the vector $\begin{pmatrix} \frac{1}{3} \\ -\frac{2}{3} \\ \frac{2}{3} \end{pmatrix}$.

(b) Find a vector parallel to the vector $\begin{pmatrix} -4 \\ 3 \\ 0 \end{pmatrix}$ which has unit length.

3. A is (0,-3,5), B is (7,-6,9) and C is (21,-12,17). Show that A, B and C are collinear stating the ratio AB:BC.

4. PQRS is a parallelogram with P(3,4,0), Q(7,6,-3) and R(8,5,2). Find the coordinates of S.

5. (a) P is the point (-1,8,0) and Q is (4,-2,5). B divides PQ in the ratio 3:2. Find the coordinates of B.

(b) A is (0,1,5) and C is (8,5,-3). Show that A, B and C are collinear.

6. An aeroplane flies in a straight line at a constant speed. It takes 3 hours to fly from A to B and 4 hours to fly from B to C.

Relative to coordinate axes, A is (0,-1,6) and C is (7,6,-1). Find the coordinates of B.

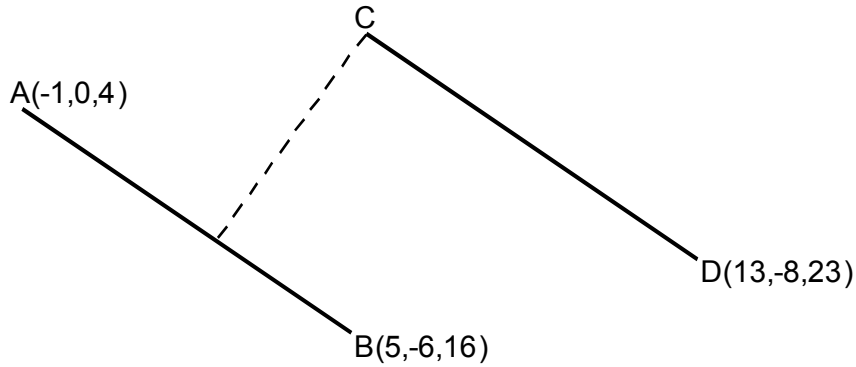


7. $\mathbf{u} = 2\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ and $\mathbf{v} = \mathbf{i} + a\mathbf{j} + \sqrt{7}\mathbf{k}$. If $|\mathbf{u}| = |\mathbf{v}|$ find the value of a.

8. Show that the vectors $\mathbf{a} = 2\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} - 7\mathbf{j} - 6\mathbf{k}$ are perpendicular.

9. A triangle has vertices A(6,-1,9), B(3,-2,11) and C(7,-8,14). Show that this triangle is right-angled at B.

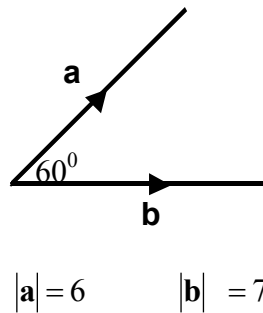
10. Three points A, B and D have coordinates as shown.



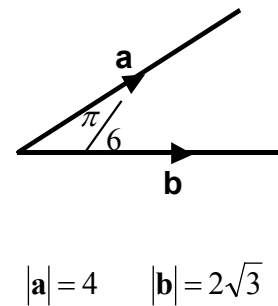
- Find the coordinates of C if AB is parallel and equal in length to CD.
- The point E divides AB in the ratio 2:1, find the coordinates of E.
- Prove that CE is perpendicular to AB.

11. Use the diagrams to find the value of $\mathbf{a} \cdot \mathbf{b}$.

(a)

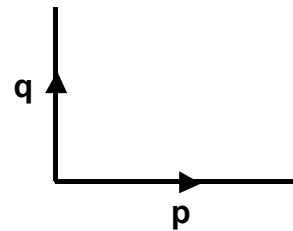


(b)



12. Write down the value of $\mathbf{p} \cdot \mathbf{q}$.

$$|\mathbf{p}| = 8 \quad |\mathbf{q}| = 9$$



13. A triangle is formed from R(0,4,-1), S(1,5,2) and T(6,1,-2).

- Find the vectors $\overrightarrow{RS}$ and $\overrightarrow{RT}$.
- Evaluate $\overrightarrow{RS} \cdot \overrightarrow{RT}$
- What can you deduce about the lines RS and RT.

14. A, B, C and D are the points (-1,3,1), (1,6,7), (0,2,5) and (1,4,10) respectively.

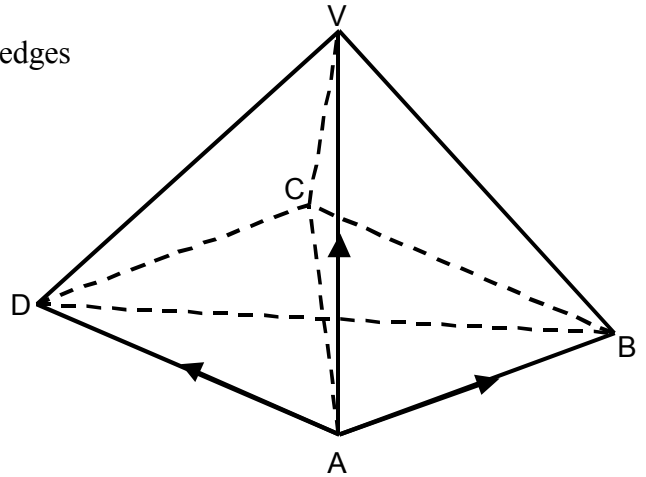
- Find the components of $\overrightarrow{AB}$ and $\overrightarrow{CD}$.
- The vector $\begin{pmatrix} p \\ q \\ 1 \end{pmatrix}$ is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{CD}$. Find p and q.

15. $\mathbf{u} = \begin{pmatrix} -3 \\ 3 \\ k \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$.

- (a) Write down the vectors $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$.
 (b) Given that $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$ are perpendicular find k .

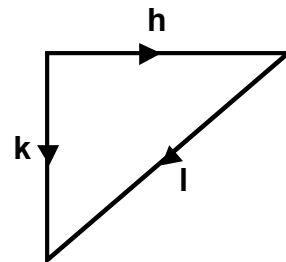
16. In the square based pyramid opposite all eight edges are of length 5 units.

Evaluate $\mathbf{p} \cdot (\mathbf{q} + \mathbf{r})$.



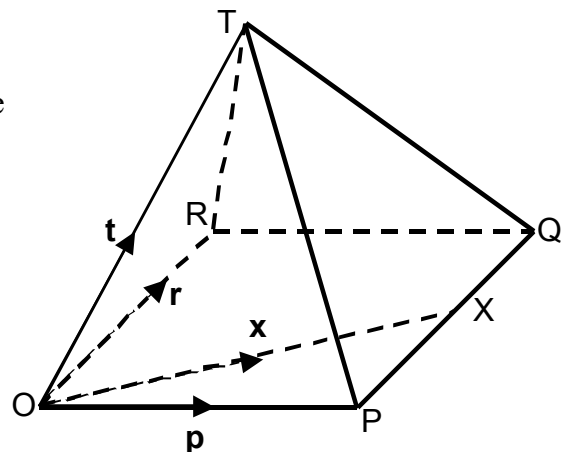
17. Shown opposite is a right-angled isosceles triangle. The two equal sides of the triangle have length 4 units.

Find the value of $\mathbf{k} \cdot (\mathbf{h} + \mathbf{k} + \mathbf{l})$.



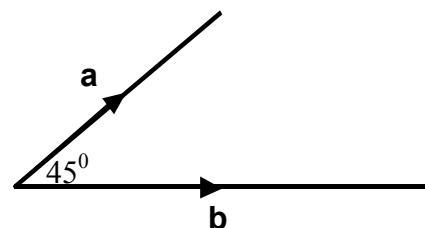
18. In the diagram opposite TOPQR is a pyramid whose base OPQR is a rhombus of length 1 unit. OPT and ORT are equilateral triangles.

- (a) Evaluate $\mathbf{t} \cdot \mathbf{r}$.
 (b) Given X is the midpoint of PQ, evaluate $\mathbf{t} \cdot \mathbf{x}$.



19. The diagram shows two vectors $\mathbf{a}$ and $\mathbf{b}$ with $|\mathbf{a}| = 2$ and $|\mathbf{b}| = 3\sqrt{3}$.

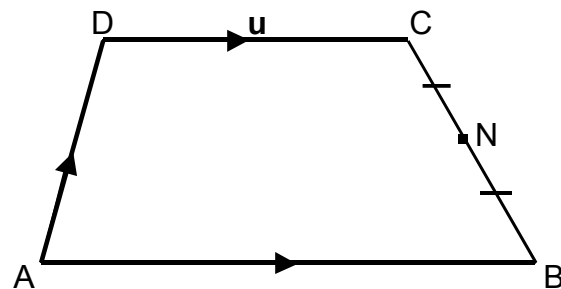
- (a) Evaluate (a) $\mathbf{a} \cdot \mathbf{a}$ (b) $\mathbf{b} \cdot \mathbf{b}$ (c) $\mathbf{a} \cdot \mathbf{b}$
 (b) Given $\mathbf{p} = 2\mathbf{a} + 3\mathbf{b}$ evaluate $\mathbf{p} \cdot \mathbf{p}$.



20. In the trapezium $AB = 2DC$ and AB is parallel to DC .

In terms of $\mathbf{u}$ and $\mathbf{v}$, write down the vectors

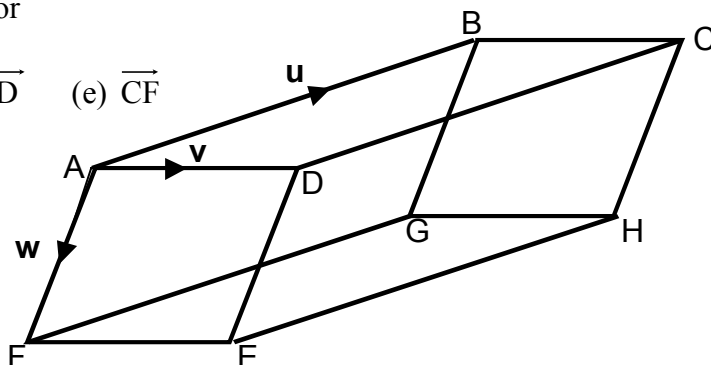
- (a) $\overrightarrow{AB}$ (b) $\overrightarrow{AC}$ (c) $\overrightarrow{BC}$ (d) $\overrightarrow{AN}$



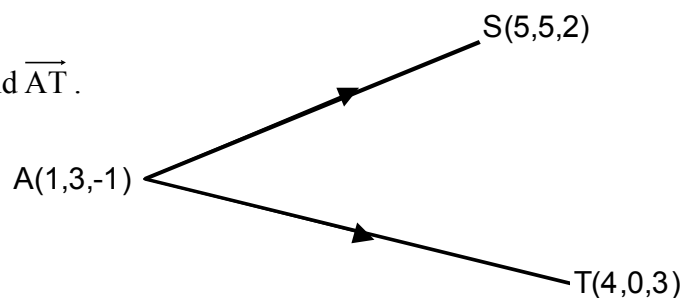
21. ABCDEFGH is a parallelepiped.

In terms of $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ find expressions for

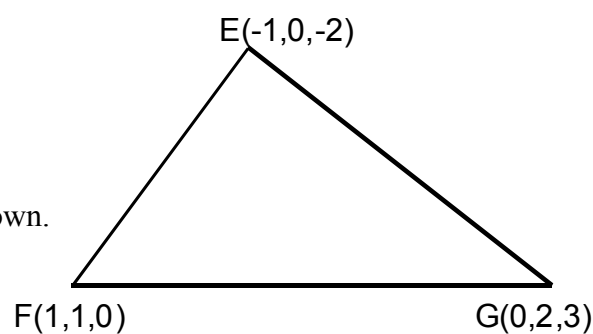
- (a) $\overrightarrow{DC}$ (b) $\overrightarrow{HC}$ (c) $\overrightarrow{AC}$ (d) $\overrightarrow{FD}$ (e) $\overrightarrow{CF}$



22. (a) For the diagram opposite find $\overrightarrow{AS}$ and $\overrightarrow{AT}$.
(b) Hence calculate angle TAS.



23. Calculate the size of angle FEG in the diagram shown.

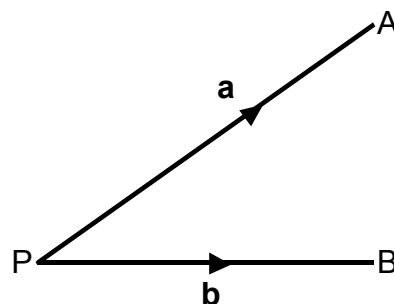


24. $\overrightarrow{PA}$ and $\overrightarrow{PB}$ are representatives of the vectors $\mathbf{a}$ and $\mathbf{b}$.

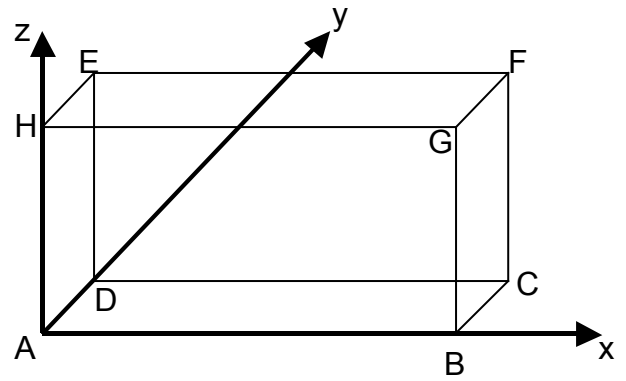
$$\mathbf{a} = \begin{pmatrix} 4 \\ -4 \\ 2 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \text{ and angle } APB = 2\theta.$$

- (a) Prove that $\cos 2\theta = -\frac{7}{9}$

- (b) Hence find the exact value of $\cos^2\theta$.



25. In the diagram $AB = 15$, $BC = 6$ and $CF = 8$
 (a) Write down the coordinates of D and F
 (b) Calculate the size of angle DBF.



26. The diagram shows three cuboids placed on top of each other.
 Two of the cuboids are equal in size –
 10 cm by 3 cm by 5 cm.
 The third cuboid is centrally placed on the
 other two and has dimensions 6 cm by 3 cm
 by 5 cm.
 (a) Write down the coordinates of A, B and C.
 (b) Calculate the size of angle BAC.

